EXPERIMENTAL STUDY OF TRANSIENT THERMOELECTRIC COOLING

III. COMPOUND MODE

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Results are shown of an experimental study concerning the compound mode of thermoelectric cooling. The test data are compared with calculations. It is shown that cooling deeper than maximum steady-state cooling can be attained.

The thermoelectric method of cooling is used more widely now in many branches of science and technology. In all applications so far, however, the thermoelectric element operates in the steady mode, which has been thoroughly analyzed [1].

Theoretical and experimental data on the transient mode of cooling (steady-state representing a special case) obtained by researchers in this and other countries [2-4] reveal new possible applications and ways to use thermoelectric cooling devices.

As we have already established experimentally in [5], the transient mode of operating thermoelectric elements offers several advantages over the steady mode, for instance: the possibility of controlling the cold-junction temperature and of deeper cooling than the maximum attainable in the steady mode (ΔT_{st}^{max}). Thus, for example, a current variation according to an extremal law ($j_{ext} = A/\sqrt{t_0 - t}$, with A and t_0 constant parameters) yields a cooling approximately 1.5 times deeper than ΔT_{st}^{max} (ΔT_{ext} = 94°C, as compared to ΔT_{st}^{max} = 66°C with $z = 2.65 \cdot 10^{-3}$/°C).

Another qualitatively novel technique of transient thermoelectric deep cooling is the compound mode [3, 4].

In the compound mode the thermoelectric element is first energized with the optimum current j_{opt} which produces a steady-state temperature distribution, and then with a current pulse which produces extra cooling of the cold junction.

The compound mode of cooling was first studied experimentally by L. S. Stil'bans and N. A. Fedorovich [6]. When a current equal to twice the optimum current was sent through a thermoelectric element operating in the steady mode (ΔT_{st}^{max} = 40°C), there resulted extra cooling by approximately 12°C. Some other results have been subsequently reported in [7].

The compound "optimum current plus rectangular pulse" mode was analyzed mathematically in [3].

We will present here the results of a thorough experimental study concerning the compound "optimum current plus rectangular pulse" and "optimum current plus extremal pulse" modes.

I. Test Results. The test procedure was similar to that followed in [8]. The compound modes were examined over a wide range of current pulses at three different initial temperatures (+50, +20, and −30°C).

1. "Optimum Current Plus Rectangular Pulse" Mode. The temperature of the cold junction as a function of time, after switching on the current in the "optimum current plus rectangular pulse" mode, is shown on an oscillogram (Fig. 1a, curve 1). Triggering the pulse at the instant t = 0 (jump of the j curve)

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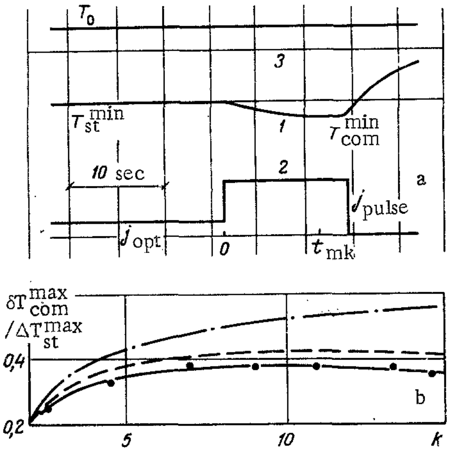

Fig. 1

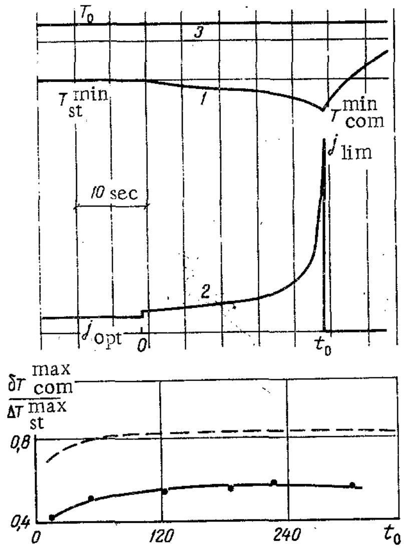

Fig. 2

Fig. 1. (a) typical oscillogram of the "optimum current plus rectangular pulse" mode: cooling curve for the cold junction of a thermoelectric element (1), current in the circuit of the thermoelectric element (2), $T_{st}^{min} = -42°C$, $T_{com}^{min} = -58°C$, $T_0 = 20°C$, $j_{opt} = 38\ A/cm^2$, $j_{pulse} = 134\ A/cm^2$, $t_0 = 10$ sec, T = 0 level (3), and maximum extra cooling of the cold junction as a function of the pulse current (in $\delta T_{com}^{max}/\Delta T_{st}^{max}$, k coordinates). b): test results (solid line), calculated results with $R_c s = 7 \cdot 10^{-5}\ \Omega \cdot cm^2$ (dashed line), calculated results with $R_c = 0$ (dashed-dotted line). Abscissa readings begin with k = 2.

Fig. 2. (a) typical oscillogram of the "optimum current plus extremal current" mode: cooling curve for the cold junction of a thermoelectric element (1), current in the circuit of the thermoelectric element (2), T = 0 level (3), $T_{st}^{min} = -46°C$, $T_{com}^{min} = -82°C$, $T_0 = 20°C$, $j_{opt} = 10\ A/cm^2$, $j_{lim} = 103\ A/cm^2$, $t_0 = 25$ sec, and maximum extra cooling of the cold junction of the pulse width t_0 (sec) (in $\delta T_{com}^{max}/\Delta T_{st}^{max}$, t_0 coordinates). b): test results (solid line), calculated results with $R_c S = 7 \cdot 10^{-5}\ \Omega \cdot cm^2$ (dashed line).

is accompanied by a smooth temperature drop from T_{st}^{min} to T_{com}^{min}, which occurs at some instant t_{mk}, and is followed by a temperature rise at the cold junction. As a rule, the current is then switched off at instant $t = t_{mk}$.

A comparison between the T(0, t) curves for various values of j shows that the cooling effect in the compound mode depends on the current flowing through the thermoelectric element. The maximum cooling as a function of the pulse current density j is shown in Fig. 1b (in $\delta T_{com}^{max}/\Delta T_{st}^{max}$, $k = j/j_{opt} \geq 2$ coordinates) [8].

As the current rises, according to Fig. 1b, δT_{com}^{max} first increases to a maximum and then decreases. Such is the trend of the curve regardless of the initial temperature T_0. However, the quantitative characteristics of cooling (location and magnitude of the $\Delta T = f(k)$ curve peak) depend somewhat on the initial temperature.

The deepest cooling attained in this experiment by the compound "optimum current plus rectangular pulse" mode (with a thermoelectric element producing $\Delta T_{st}^{max} = 66°C$ at $T_0 = +20°C$, $z = 2.56 \cdot 10^{-3}/°C$) was 90° ($1.36\ \Delta T_{st}^{max}$ at k = 9), comparable with the capability of modern two-stage thermo batteries with the number of elements per stage in the ratio 1 : 10.

2. "Optimum Current Plus Extremal Current" Mode. It has been shown in [9] that maximum transient cooling can be attained when the current rises according to an extremal law. In view of this, we studied also the "optimum current plus extremal current" mode. When extremal current (j_{ext}) flows through a thermoelectric element operating in the steady mode at j_{opt}, then the extra cooling of the cold

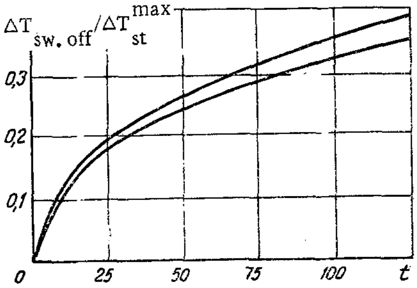

Fig. 3. Temperature of the cold junction in a thermoelectric element, as a function of time after the optimum current has been switched off: measured values (a), calculated values (b). Time t (sec).

junction (Fig. 2a) is much deeper than δT_{com}^{max} with a rectangular pulse. As in the case of an isolated extremal current pulse (without preliminary cooling of the cold junction by the optimum current), most of the cooling occurs at the end of the current period t_0. A comparison between the cooling test curves (Fig. 2b) shows that, as t_0 becomes longer, the maximum cooling first increases to a peak and then decreases. In this case, however, the peak is wider and softer than at j = const. The deepest cooling is 1.5-1.6 times ΔT_{st}^{max}.

In this mode we attained a temperature drop ΔT_{ext}^{max} = 104°C, near the capability of modern three-stage thermo batteries.

A higher initial temperature, according to our experiment, results (regardless of the pulse shape) in extra cooling, while a lower T_0 results in a smaller δT_{com}. Thus, raising the initial temperature by 80°C (from −30 to +50°C) was in our experiment followed by a 10°C larger δT_{com} with an extremal current pulse and by a 9°C larger δT_{com} with a rectangular current pulse.

II. Discussion of the Results. Our test data will be evaluated here on the basis of the model which was used earlier in [8]: the parameters of both branches (ρ, $\varkappa$, a, Π, l, S) are considered respectively identical and independent of the temperature, the space coordinates, and time, the lateral surfaces are thermally insulated, no load is applied to the cold junction, and the Thompson heat is negligible.

The optimum current flowing through a thermoelectric element produces in it a steady temperature distribution, on which another temperature distribution due to the current pulse is then superposed.

Since the properties of the element material are considered independent of the temperature, hence one may apply here the superposition principle and thus considerably simplify the analysis of the process. It is assumed, moreover, that simultaneously at the time t = 0 the optimum current is switched off and the current pulse is applied. The cooling of the cold junction in the compound mode is thus governed by two independent factors: a "warming" of the junction $\Delta T_{sw.off}$ (due to a diffusion of the steady-state temperature distribution after the optimum current has been switched off) and a cooling ΔT_{pulse} caused by the current pulse:

$$\Delta T_{com} = \Delta T_{st}^{max} + \delta T_{com}, \tag{1}$$

with δT_{com} denoting the amount of extra cooling

$$\delta T_{com} = \Delta T_{pulse} - \Delta T_{sw.off}. \tag{1a}$$

In order to determine $\Delta T_{sw.off}$, it is necessary to solve the equation of heat conduction $\partial T/\partial t = a(\partial^2 T/\partial x^2)$ for the boundary condition $\partial T/\partial x = 0$ at $x = 0$ and the initial condition $T(x, 0)$.

It is not difficult to show that $T(x, 0) = T_H - \Delta T_{st}^{max}(b^2/(b+1)^2)(1 - x/l)^2$, with T_H denoting the temperature of the hot junction ($T_H = T_0$ = const.), $b = 2\rho l/R_c S$, and R_c denoting the contact resistance of the cold junction. The formula for the temperature distribution in a half-space with an adiabatic insulated boundary [10] yields

$$T(x, t) = \frac{1}{2\sqrt{\pi a t}} \int_0^\infty T(\xi, 0)\left\{\exp\left[-\frac{(x+\xi)^2}{4at}\right] - \exp\left[-\frac{(x-\xi)^2}{4at}\right]\right\} d\xi$$

and, inserting here the expression for $T(\xi, 0)$, we have

$$\Delta T_{sw.off} = 2\Delta T_{st}^{max} \frac{b^2}{(b+1)^2}\left(\frac{2}{\sqrt{\pi}} - \frac{\sqrt{at}}{l}\right)\frac{\sqrt{at}}{l}. \tag{2}$$

The measured "warming" curve for a cold junction if compared in Fig. 3 with the curve calculated according to formula (2). Taking into account that the model of a thermoelectric element is here an approximation only, one must consider the agreement between both curves to be satisfactory.

Compound Cooling

a) Rectangular Pulse. The cooling of a cold junction, when a rectangular current pulse flows through it, can be described by the expression [8]:

$$\Delta T_{st}^{max} = 2\Delta T_{pulse} \frac{bk}{(b+1)^2 l} \left[\frac{2}{\sqrt{\pi}} (b+1-k) \sqrt{at} - \frac{bk}{l} at \right]. \tag{3}$$

Inserting (2) and (3) into (1a), then differentiating the new expression with respect to t and equating the derivative to zero, we obtain an equation which relates the current j and the time t_{mk} to reach maximum cooling in the compound mode as follows:

$$\rho \sqrt{\pi a t_{mk}} = \frac{\Pi}{j} \cdot \frac{(b-k)k}{(b+1)(k+1)}. \tag{4}$$

Inserting (2), (3), and (4) into (1a) yields the maximum extra cooling in the compound mode:

$$\delta T_{com}^{max} = \frac{2}{\pi} \Delta T_{st}^{max} \frac{k-1}{k+1} \left(\frac{b-k}{b+1} \right)^2. \tag{5}$$

When $R_c = 0$, formula (5) becomes

$$\delta T_{com}^{max} = \frac{2}{\pi} \Delta T_{st}^{max} \frac{k-1}{k+1}. \tag{6}$$

According to (6), the extra cooling must become deeper as the current is increased, approaching $(2/\pi)\Delta T_{st}^{max}$ when $k \to \infty$.

The physical significance of this conclusion is as follows. According to formula (2), the preliminary cooling effect produced by the flow of the optimum current will diminish with time after this current has been switched off. This means that compound cooling is most effective during the initial period after the optimum current has been switched off. In order to produce as much cooling as possible, therefore, it is necessary to apply shorter current pulses (i.e., pulses yielding the maximum cooling within a sufficiently short time t_m while the cold junction has "warmed up" by a still negligible amount with the optimum current switched off). Since the time t_m to reach maximum cooling by a pulse is related to the current, namely $t_m \sim 1/j^2$ [8], hence a short time of maximum preliminary (steady-state) cooling must correspond to a heavy current ($j \gg j_{opt}$, $k \gg 1$).

This analysis remains valid, however, only as long as a negligible quantity of Joule heat is generated across the junction contact resistance, which is true in the case of light currents (i.e., long t_m periods) but not in the case of heavy currents, when, according to Eq. (5), the amount of cooling is diminished and the preliminary cooling is thus not fully utilized.

Thus, the compound mode of operation of a thermoelectric element must cover two ranges of current density: 1) the light-current range (long t_{mk} periods), where the extra cooling δT_{com} is slight because of an appreciable "warming" of the junction, and 2) the heavy-current range (short t_{mk} periods), where the extra cooling δT_{com} is slight because of Joule heat generation in the junction. This conclusion agrees with the results of our experiment (Fig. 1b).

In Fig. 1b are shown curves calculated according to formulas (5) and (6). According to this graph, the theoretical curve which takes into account the contact resistance R_c (dashed line) follows the same trend as the curve based on tests. The position of the maximum point k_{opt} is easily shown to be related to the parameter b as follows:

$$k_{pulse} \approx \sqrt{b}. \tag{7}$$

For a thermoelectric element with a curve as in Fig. 1b, the calculated value $k_{opt} \approx 11$ ($R_cS = 7 \cdot 10^{-5}$ $\Omega \cdot cm^2$, $l = 6.5$ cm, $\rho = 6.6 \cdot 10^{-4}$ $\Omega \cdot$cm, $b = 120$) is sufficiently close to the test value $k_{opt} = 9$. As to the magnitude of δT_{com}^{max}, the calculated extra cooling is deeper than the measured cooling for all values of k.

In the case of $R_c = 0$, as was to be expected, the theoretical values of δT_{com}^{max} (dashed-dotted line) are much higher than the test values, which indicates the necessity of considering the contact resistance in an analysis of the compound cooling mode.

b) Extremal Current. As has been shown in [9], the maximum cooling effect with a current pulse alone is obtained when the latter is extremal:

$$j_{ext} = \frac{\Pi}{2\rho\sqrt{\pi a(t_0 - t)} + R_c S}. \tag{8}$$

*In the derivation of relation (7) one assumes that $b \gg 1$; in actual thermoelectric elements $b \approx 50$-300.

Here t_0 denotes the pulse width ($t_0 \ll l^2/a$) and t is the time of the test. The expression for ΔT_{ext}^{max} becomes here

$$\Delta T_{ext}^{max} = \Delta T_{st}^{max} \frac{1}{\pi} \ln\left(\frac{2\rho\sqrt{\pi a t_0}}{R_c S} + 1\right). \tag{9}$$

Using the superposition principle and inserting (9) and (2) into (1a), we obtain an expression for the deepest extra cooling in the compound "optimum current plus extremal current" mode, as a function of t_0:

$$\delta T_{com}^{max} = \Delta T_{st}^{max} \frac{1}{\pi} \ln\left(\frac{2\rho\sqrt{\pi a t_0}}{R_c S} + 1\right)$$
$$-\Delta T_{st}^{max} \frac{2}{\pi} \cdot \frac{b^2}{(b+1)^2}\left(2 - \frac{\sqrt{\pi a t_0}}{l}\right)\frac{\sqrt{\pi a t_0}}{l}. \tag{10}$$

In Fig. 2b (dashed line) is shown the ratio $\delta T_{com}^{max}/\Delta T_{st}^{max} = f(t_0)$ plotted according to formula (10). As in the case of a rectangular pulse, b = 120, l = 6.5 cm, and $a = 12 \cdot 10^{-3}$ cm²/sec here.

A comparison between the curves in Fig. 2b indicates that formula (10) correctly describes the qualitative trend of the test curve. Just as the test curve, the theoretical curve too has a peak whose magnitude and position in time can be found from expression (10). Considering that $b \gg 1$, we have

$$t_{om} = \frac{l^2}{4\pi a}. \tag{11}$$

Inserting (11) into (10) yields

$$\delta T_{m\,com}^{max} = \Delta T_{st}^{max}\left[\frac{1}{\pi} \ln b - 0.7\right]. \tag{12}$$

For our particular thermoelectric element we calculate

$$t_{om} = 276\,\text{sec}, \quad \delta T_{m\,com}^{max} = 0.84 \Delta T_{st}^{max}.$$

The calculated maximum is shifted from the measured one toward somewhat longer time periods t_0.

It is to be noted that the calculations made here are not rigourously quantitative, inasmuch as a number of factors has not been taken into account. The adequate agreement between experimental and theoretical data, however, makes the selected approximate model, which accounts only for the absorption of Peltier heat and the generation of Joule heat in the volume and in the junction of a thermoelectric element, acceptable for describing the performance of a thermoelectric element in the compound mode.

This approximate model does not explain the dependence of δT_{com}^{max} on T_0. As has been shown in [3] for the compound "optimum current plus rectangular pulse" mode, however, taking into account the temperature-dependence of the Peltier coefficient ($\Pi = \alpha T$) will make the maximum extra cooling in the compound mode a function of the initial temperature. The results obtained in such an analysis agree with test data.

NOTATION

α	is the coefficient of thermo emf;
ρ	is the electrical resistivity;
$\varkappa$	is the thermal conductivity;
Π	is the Peltier coefficient;
a	is the thermal diffusivity;
z	is the thermoelectric effectiveness;
l	is the length of thermoelectric element;
S	is the cross section area of thermoelectric element;
j	is the current density;
j_{opt}	is the optimum current density;
j_{ext}	is the current density in the extremal mode;
k	is the ratio of current density in the compound mode to optimum current density;
R_c	is the contact resistance of the cold junction;
t	is the time;
t_m	is the time to reach lowest temperature in the j = const. mode;

t_0 is the pulse width in the extremal mode;
t_{mk} is the time to reach deepest cooling in the compound "optimum current plus rectangular pulse" mode;
t_{0m} is the time to reach deepest cooling in the compound "optimum current plus extremal current" mode;
T_0 is the initial temperature of thermoelectric element;
T_x is the temperature of the cold junction;
T_{st}^{min} is the lowest steady-state temperature of the cold junction;
ΔT_{pulse} is the cooling effect of a pulse;
$\Delta T_{sw.off}$ is the "warming" of the cold junction;
δT_{com} is the extra cooling in the compound mode;
δT_{com}^{max} is the deepest extra cooling in the compound mode.

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